

# Advanced Quantitative Methods

## Regression Discontinuity Designs

Instructor: Gregory Eady  
Office: 18.2.10  
Office hours: Fridays 13-15

# Today

- Course evaluations
- Introduction to Regression Discontinuity Designs (RDD)
- Regression discontinuity exercise in R

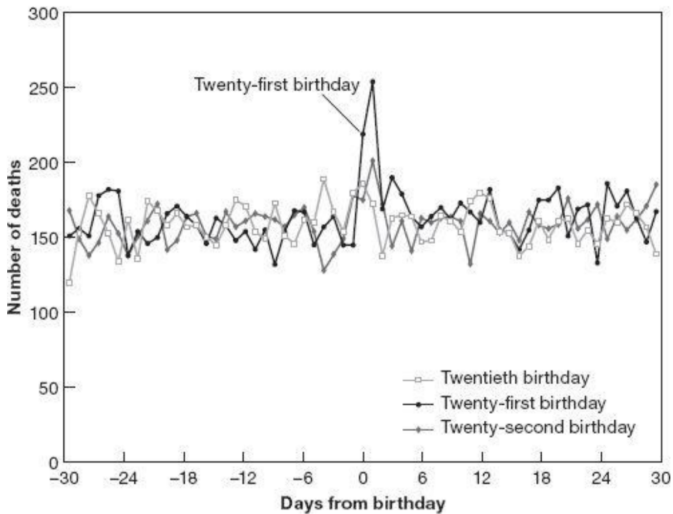
# Course evaluations

- <https://evaluating.ku.dk/>

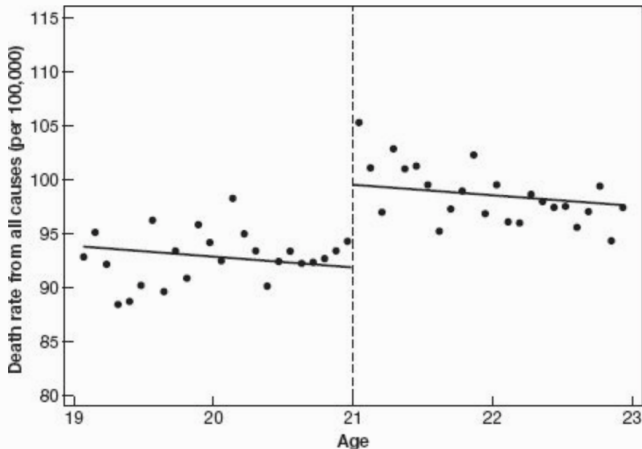
## Regression discontinuity designs

- Human behavior is often constrained by rules:
  - Elections are won by candidates with the most votes
  - University courses are capped at a certain class size
  - Only certain people are eligible for citizenship
- RDDs exploit these types rules to estimate a causal effect
- Conceptually, RDDs are very intuitive

FIGURE 4.1  
Birthdays and funerals



## A sharp RD estimate of MLDA mortality effects



MLDA: Minimum legal drinking age

## Basic setup

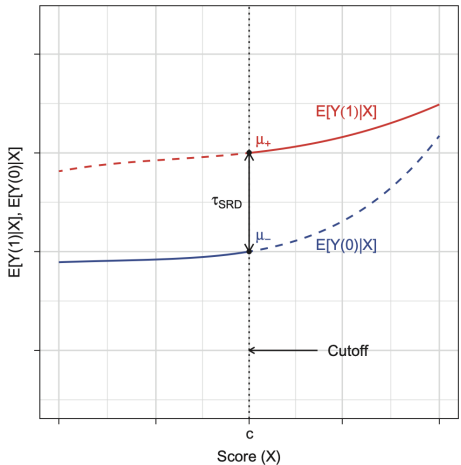
Treatment indicator:

$$T_i = \begin{cases} 1 & \text{if } x_i \geq x_0 \\ 0 & \text{if } x_i < x_0, \end{cases} \quad (1)$$

where  $x_i$  is the “running variable,” and  $x_0$  denotes the value of the running variable at the cutoff.

The key assumption concerns continuity: that potential outcomes for units at the threshold (when  $x_i = x_0$ ) are continuous, such that any discontinuity in the outcome can be attributed to the treatment effect.

# What are we estimating?



**Figure 2** RD Treatment Effect  $\mu_+$  in the Sharp RD Design



## RDD estimates are LATE (local average treatment effects)

“Regression discontinuity identifies effects *local* to the relevant cut points, ... [just as] experiments identify effects local to the typically non-representative sample of experimental subjects.” (Samii 2016)

## The basic setup to estimate an RDD effect:

$$y_i = \alpha + \beta x_i + \rho T_i + \eta_i, \quad (2)$$

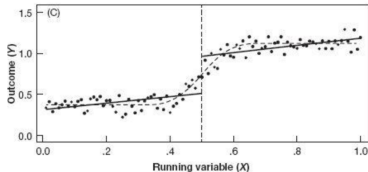
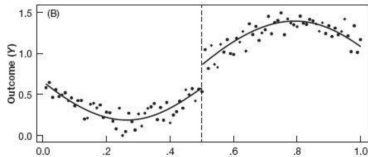
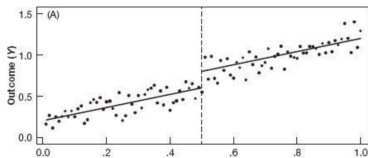
where  $y_i$  is a continuous function of  $x_i$  except for at the discontinuity at  $x_0$ .

The magnitude of this effect is given by the estimate of  $\rho$ .

## RDD is simple in principle, but many potential problems can arise

1. The functional form of the regression model does not sufficiently captures non-linearities in the running variable
2. The potential outcomes are not smooth across the discontinuity (i.e. “sorting”)

## Functional form



## How do we deal with these problems? We run a ton of sensitivity analyses

### 1. Higher-order polynomial to model the non-linearity

- e.g.  $y_i = \alpha + \rho T_i + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i T_i + \beta_4 x_i^2 T_i$
- Make sure to transform  $x_i$  so it is centered on the cutoff
  - Remember how interaction terms work!
  - Given the interaction terms, parameter  $\rho$  represents the relationship between  $T_i$  and the outcome when  $x_i = 0$ . So we want to be sure that  $x_i = 0$  is the cutoff!

### 2. Local linear regression with a small bandwidth around the cutoff

- The “bandwidth” indicates how much data around the cutoff we use in our analysis
- Why? The problem is that we don't know the correct functional form. When we zoom in on data closer to the cutoff, the function will be increasingly linear...

# Decreasing the bandwidth limits assumptions about functional form:

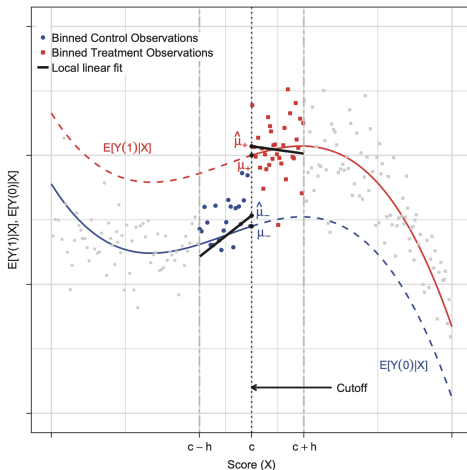


Figure 12 RD Estimation with Local Polynomial

## Potential “sorting” close to the cutoff

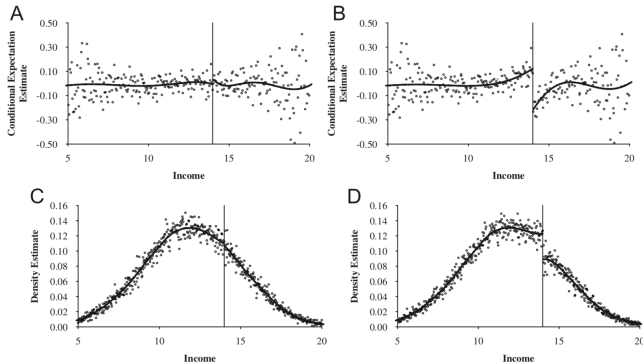


Fig. 2. Hypothetical example: gaming the system with an income-tested job training program: (A) conditional expectation of returns to treatment with no pre-announcement and no manipulation; (B) conditional expectation of returns to treatment with pre-announcement and manipulation; (C) density of income with no pre-announcement and no manipulation; (D) density of income with pre-announcement and manipulation.

Example is for an income-tested job program

## No “sorting” problem in House elections:

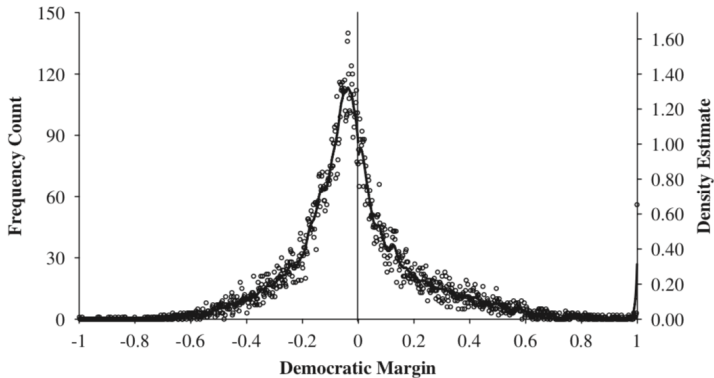


Fig. 4. Democratic vote share relative to cutoff: popular elections to the House of Representatives, 1900–1990.



## “Sorting” in roll-call voting:

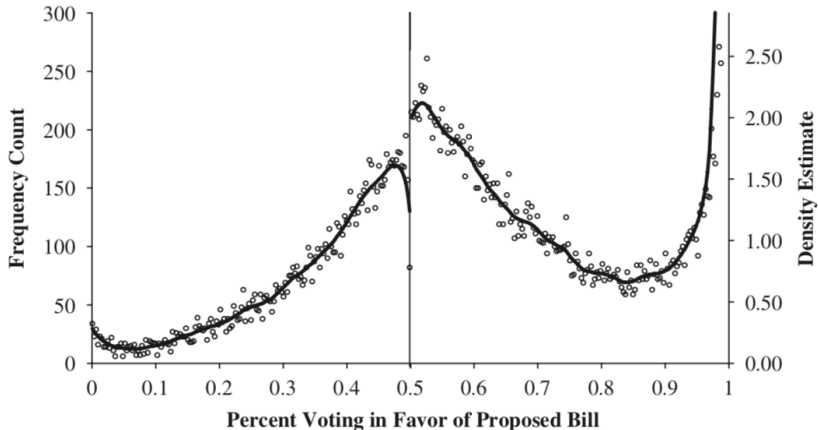


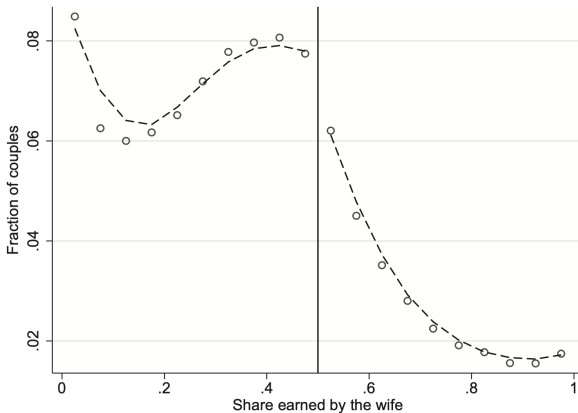
Fig. 5. Percent voting yeay: roll call votes, U.S. House of Representatives, 1857–2004.

## Test for sorting at the threshold of the running variable

- Apply the density test proposed by McCrary (2008)
- In R, use the `DCdensity()` function from the `rdd` library

## Sorting can itself be substantively interesting

“Gender Identity and Relative Income Within Households” (Bertrand et al., 2015)



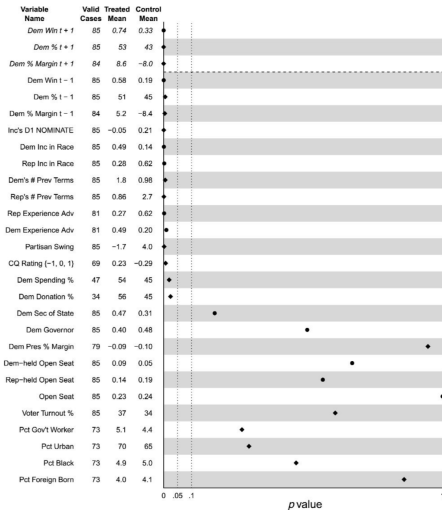
“We show that the distribution of the share of income earned by the wife exhibits a sharp drop to the right of  $\frac{1}{2}$ , where the wife’s income exceeds the husband’s income. **We argue that this pattern is best explained by gender identity norms, which induce an aversion to a situation where the wife earns more than her husband.**” (Bertrand et al., 2015)

## Also test for discontinuities in pre-treatment variables at the cutoff

- Why? Because if there is no sorting, we should not expect to see any discontinuities for background characteristics
- E.g. Sorting in US House elections...

## Elections and the Regression Discontinuity Design (Caughey and Sekhon, 2011)

“**Why are some candidates able to eke out narrow victories?** Even in competitive elections, U.S. House candidates are not evenly matched. Partisan tides may make the out-party candidate more competitive than usual, but our data show that the incumbent party’s candidate nearly always has more political experience and more money. These observable factors are likely correlated with other unobserved advantages, such as party organization, political skill, or the preferences of constituents. **In the closest elections, candidates have every incentive to make maximal use of their resources, and not coincidentally, almost three-quarters of razor-close elections break towards the party that already holds the seat.**”



**Fig. 2** Covariate balance between treated (Democratic win:  $n = 43$ ) and control (Democratic loss:  $n = 42$ ) in a 0.5% window. The first three variables listed are posttreatment outcome variables. The  $p$  values for dichotomous variables (circles) are from Fisher's exact test. Exact Wilcoxon rank sum tests were used for continuous and ordinal variables (diamonds). All  $p$  values are two-sided. Calculations are based on all cases with non-missing values for the variable.

## Extremely close races (within 0.5%) are predicted by:

- Candidates with the most money
  - Win close races 2/3 of the time
- Those with more political experience
  - Win 70% of the time
- Incumbency
  - 72% of the time

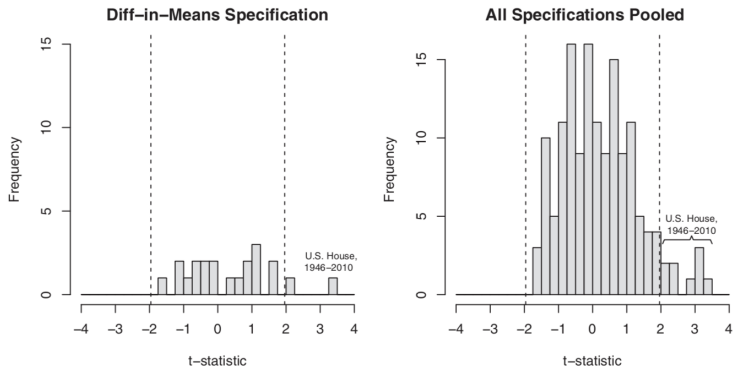


## Caughey and Sekhon (2011):

“The RD design is a powerful inferential tool that is appropriate in many situations, potentially including many elections. But the applicability of the design cannot be assumed; it must be justified on the basis of context-specific theory and data.”

## Fortunately, all is not lost with close election RDDs

FIGURE 2 T-values for “Effect” of Party Winning at Time  $t$  on Party Winning at Time  $t - 1$

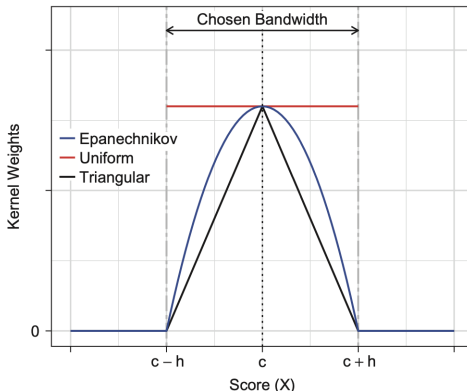


In elections data, incumbent party imbalance seems specific U.S. House elections, 1946-2010 (Eggers 2015)

## Sensitivity analyses in summary:

1. Test for sorting of the running variable at the cutoff (McCrary 2008)
2. Test for the absence of discontinuities in pre-treatment outcomes that might cause sorting
3. Test for sensitivity of results to different functional forms (linear, quadratic, cubic, quartic, etc.)
4. Test for sensitivity of results to different bandwidths
5. And finally, test with local linear regression using the “optimal bandwidth” (Imbens & Kalyanaraman, 2011)
  - 5.1 Local linear regression (with a rectangular kernel for weighting)
  - 5.2 Select bandwidth to minimize the mean-squared error

## Kernel weighting



**Figure 13** Different Kernel Weights for RD Estimation

A triangular kernel for weighting observations, and a polynomial of order 1 (i.e. local linear regression) is the standard

## Basic idea behind optimal bandwidth selection (Imbens & Kalyanaraman, 2011)

“[I]f the window is very narrow, there are few observations left, meaning the resulting estimates are likely to be too imprecise to be useful. Still, we should be able to trade the reduction in bias near the boundary against the increased variance suffered by throwing data away, generating some kind of optimal window size.” (Angrist & Pischke 2014)

## Working through a well-known applied example

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### What Happens When Extremists Win Primaries?

ANDREW B. HALL *Harvard University*

**T***his article studies the interplay of U.S. primary and general elections. Examine how the nomination of an extremist changes general-election outcomes and legislative behavior in the U.S. House, 1980–2010, using a regression discontinuity design in primary elections. When an extremist—as measured by primary-election campaign receipt patterns—wins a “coin-flip” election over a more moderate candidate, the party’s general-election vote share decreases on average by approximately 9–13 percentage points, and the probability that the party wins the seat decreases by 35–54 percentage points. This electoral penalty is so large that nominating the more extreme primary candidate causes the district’s subsequent roll-call representation to reverse, on average, becoming more liberal when an extreme Republican is nominated and more conservative when an extreme Democrat is nominated. Overall, the findings show how general-election voters act as a moderating filter in response to primary nominations.*

## What happens when extremists win primaries?

- Are both primary & general elections in the US
- Primary voters thought to generally prefer more extreme candidates (moderates often fear “getting primaried”)
- But there is a tradeoff for primary voters:
  1. They can vote in the primary for a candidate who is closer to their own (extreme) views
  2. But that candidate may then be less likely to win the general election

## The regression discontinuity design

- How does one examine the effect of electing an extreme candidate?
- Extremists and moderates who are elected will likely differ for many other reasons (candidate, challenger, and district-level confounders)
- The RDD:
  - Compare general election results for (1) moderate candidates who barely won a primary election against an extremist to (2) extremist candidates who barely won a primary against a moderate



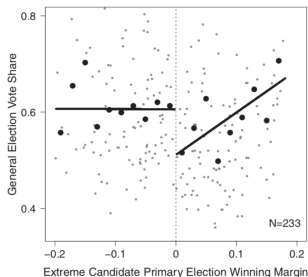
## Estimation Strategy: Regression Discontinuity Design in Primary Elections

I estimate equations of the form

$$Y_{ipt} = \beta_0 + \beta_1 \text{Extremist Primary Win}_{ipt} + f(V_{ipt}) + \epsilon_{ipt}, \quad (1)$$

## Effect of nominating an extremist on vote share

**FIGURE 2. General-Election Vote Share After Close Primary Elections Between Moderates and Extremists: U.S. House, 1980–2010**

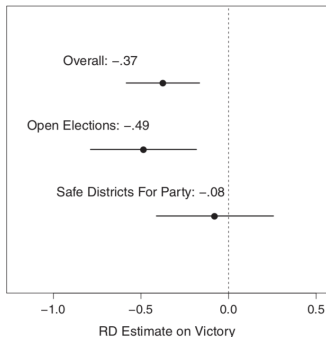


*Notes:* The close election of the more extreme primary candidate causes a decrease in general-election vote share for the party. Large black points are averages in 0.02 point bins of the relatively extreme candidate's winning margin; small gray points are raw data. Lines are OLS fits from raw data estimated separately on each side of threshold. Average general-election vote shares are above 0.5 on both sides of the discontinuity because contested primaries are more likely to occur in districts where the normal vote is tilted towards the party.

Notice the “binned” points to clarify the linear relationship

## Effect on probability of victory

**FIGURE 3. Effects of Nominating the Extremist Candidate on General Election Victory Across Primary Types**



*Notes:* The penalty to extremists is largest in primaries for open-seat general election races, and close to zero in primaries for districts that are safe for the party. Estimates are calculated according to [Equation 1](#), using the full data and a third-order polynomial of the running variable. Horizontal lines are 95% confidence intervals from robust standard errors.

## Sensitivity of results to non-linearities (linear, cubic, “optimal”) & bandwidth size

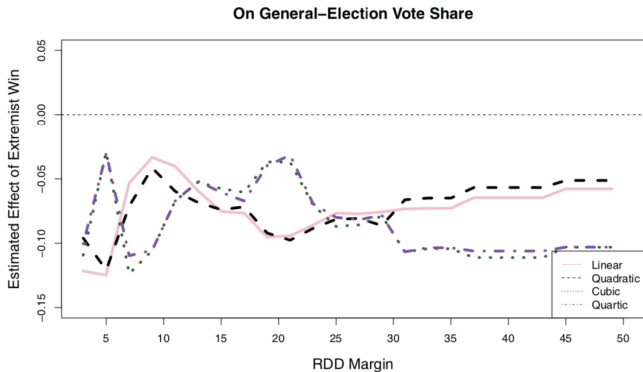
**TABLE 2. RDD Estimates of the Effect of Nominating an Extreme Candidate on General Election Vote Share, U.S. House 1980–2010**

	Vote Share General Election	Vote Share General Election	Vote Share General Election	Victory General Election	Victory General Election	Victory General Election
Extremist win	−0.12 (0.06)	−0.10 (0.03)	−0.08 [0.04]	−0.53 (0.22)	−0.37 (0.11)	−0.35 [0.17]
<i>N</i>	83	252	135	83	252	148
RDD bandwidth	5	–	8.51	5	–	9.68
Specification	Local linear	Cubic	IK	Local linear	Cubic	IK

*Notes:* Maximum of robust and conventional standard errors in parentheses. Columns 3 and 6 use optimal bandwidth technique from Imbens and Kalyanaraman, implemented using `rddob` in Stata. Standard errors from this procedure in brackets.

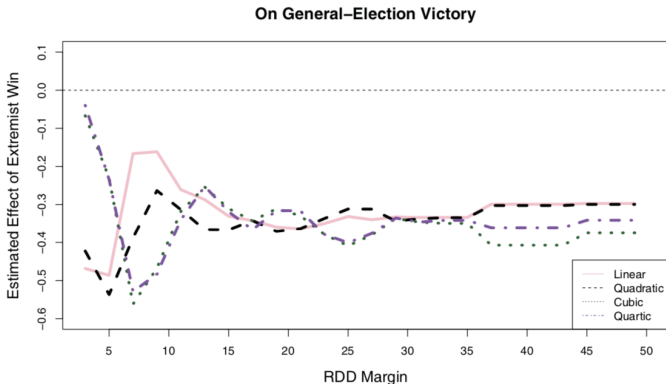
## Sensitivity of vote share results to non-linearities & bandwidths

FIGURE A.3. RDD Estimate for General-Election Vote Share Across Bandwidths from 3 to 50



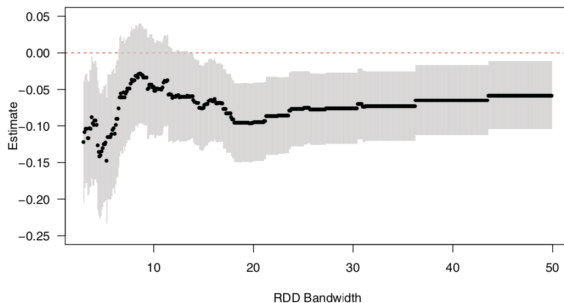
## Sensitivity of win probability results to non-linearities & bandwidths

FIGURE A.4. RDD Estimate for General-Election Victory Across Bandwidths from 3 to 50



## Sensitivity of vote share results to bandwidth selection

**FIGURE A.5. Local Linear RDD Estimate for General-Election Vote Share Across Bandwidths from 3 to 50**



## Sensitivity of win probability results to bandwidth selection

FIGURE A.6. Local Linear RDD Estimate for General-Election Victory Across Bandwidths from 3 to 50

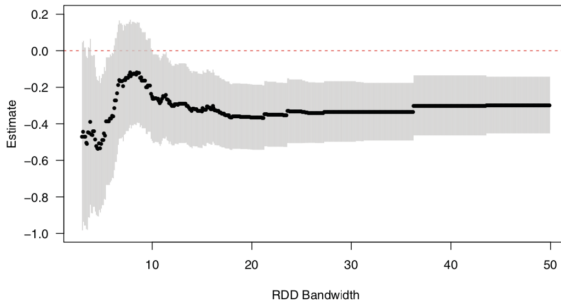
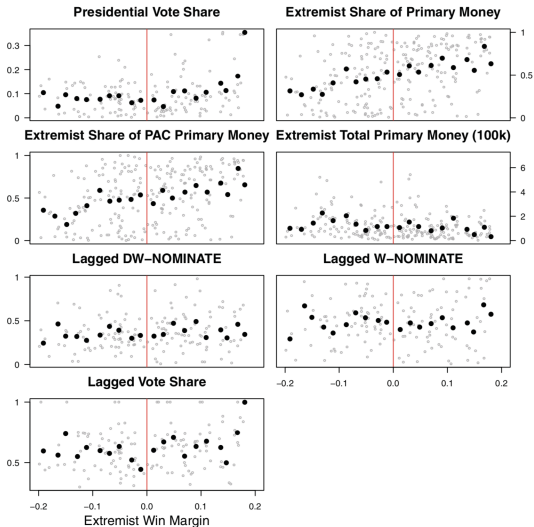




FIGURE A.2. Graphical Balance Tests



**TABLE A.5. RDD Balance Tests**

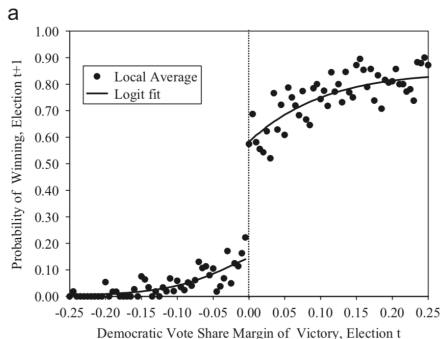
	Bandwidth Size			Adjusted $p$ value
	5%	IK	–	
Presidential normal vote, Absolute distance from 50%	–0.002 (0.049)	–0.009 [0.013]	–0.045 (0.027)	0.10
Extremist share of primary donations	–0.013 (0.154)	0.033 [0.100]	–0.007 (0.071)	0.92
Extremist share of PAC Primary donations	–0.131 (0.157)	–0.043 [0.099]	–0.092 (0.073)	0.21
Extremist total primary donations, \$100,000s	–0.181 (0.420)	0.037 [0.411]	0.138 (0.189)	0.47
Previous inc DW-NOM score, Absolute value	–0.010 (0.130)	0.038 [0.052]	–0.008 (0.071)	0.91
Previous inc W-NOM score, Absolute value (year adjusted)	0.120 (0.212)	–0.053 [0.043]	–0.167 (0.096)	0.09
Lag vote share	0.074 (0.141)	–0.091 [0.033]	0.050 (0.068)	0.46
Lag victory	0.011 (0.392)	0.005 [0.106]	–0.046 (0.188)	0.80

*Notes:* Maximum of robust and conventional standard errors in parentheses; standard errors from Imbens-Kalyanaraman in brackets. Column 1 reports results for local linear OLS estimated separately on each side of the discontinuity. Column 2 reports results using the Imbens-Kalyanaraman optimal bandwidth, implemented using `rdob` in Stata. Column 3 reports results using the full data with a cubic polynomial of the running variable. Column 4 reports  $p$  values for the cubic polynomial tests adjusted for multiple testing using Free Step-Down Resampling.

**Some RDD examples to get you thinking about RDD applications in your own research...**

## Incumbency → vote share

“Randomized experiments from non-random selection in U.S. House elections” (Lee 2008)

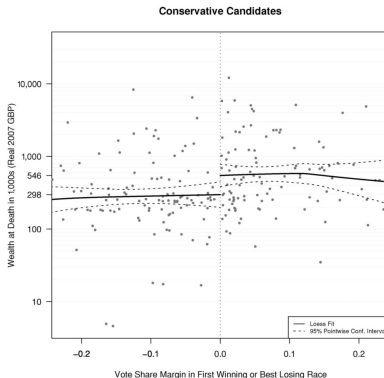


$T_i$ : (Barely) winning an election

$Y_i$ : Vote share in next election ( $\uparrow$ )

## Holding office → \$\$\$

“MPs for Sale?” (Eggers & Hainmueller, 2009)

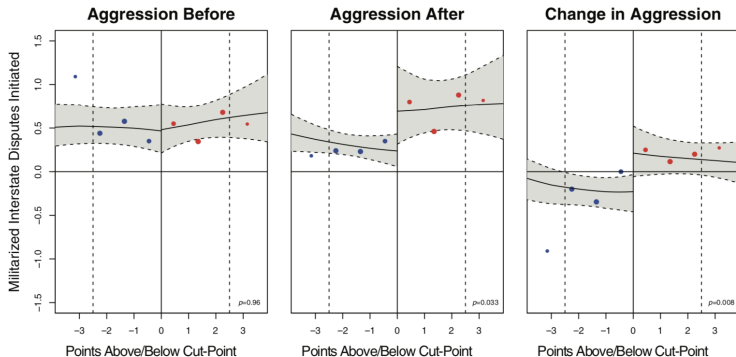


$T_i$ : (Barely) winning an election

$Y_i$ : Wealth ( $\uparrow$ )

## Nationalism → War

“Nationalism and Conflict” (Bertoli, 2017)

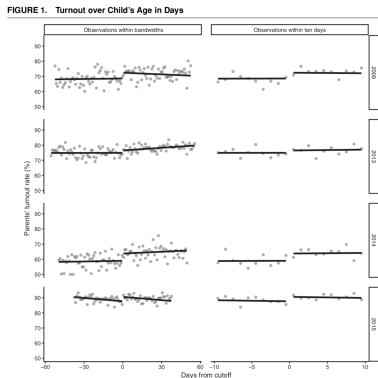


$T_i$ : (Barely) making it through the World Cup qualifiers

$Y_i$ : Interstate disputes (↑)

## Children voting → Parents voting

“Trickle-up Political Socialization” (Dahlgard, 2018)

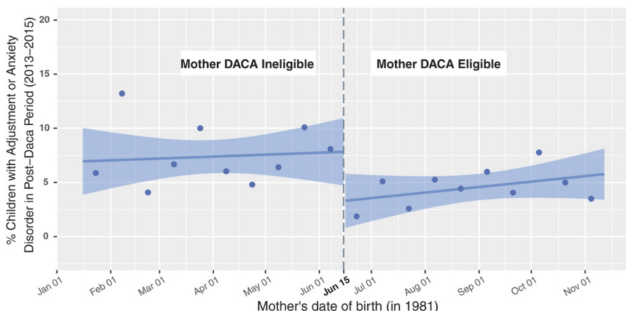


$T_i$ : Having a child (barely) of age to vote

$Y_i$ : Parental voter turnout (↑)

## Migration safety for mothers → Child mental health

“Protecting unauthorized immigrant mothers improves their children’s mental health” (Hainmueller et al., 2017)



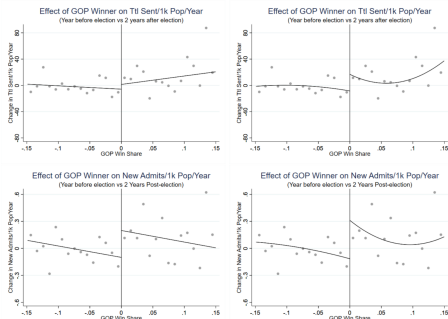
$T_i$ : Children having a mother (barely) of age to be given Deferred Action for Childhood Arrival (DACA) status

$Y_i$ : Children’s mental health (↑)



## Partisanship of district attorneys → prison admissions

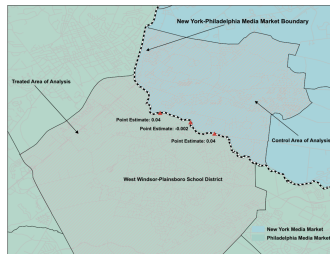
“The Effect of District Attorneys on Local Criminal Justice Outcomes” (Krumholz, 2019)



$T_i$ : Republican district attorney (barely) winning an election  
 $Y_i$ : Prison admissions ( $\uparrow$ ) & crime and arrest rates (null)

## Presidential ads → turnout

“Geographic Boundaries as Regression Discontinuities” (Keele and Titiunik, 2015)



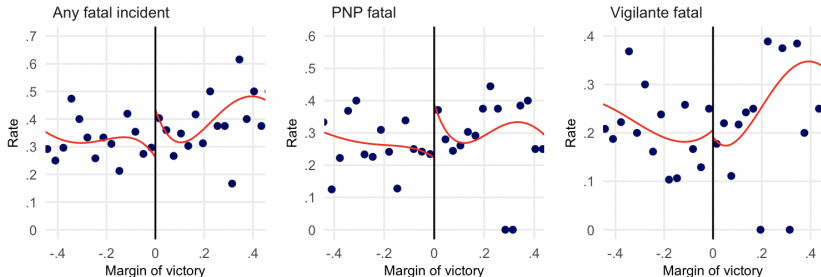
**Fig. 6** Geographically located estimated advertisement effects on 2008 voter turnout. Treatment effects estimated at three different points along the boundary between the Philadelphia, PA, media market (located southwest of the boundary) and New York City, NY, media market (located northeast of the boundary) in the state of New Jersey (see Fig. 4). Results estimated with local linear regression with triangular kernel weights on each observation's chordal distance to the point of estimation and bandwidths fixed at 1.1 km. Area marked is the West Windsor-Plainsboro school district, which straddles the media market boundary. Empirical analysis is confined to the West Windsor-Plainsboro school district only, where legislative districts are also the same on both sides of the border. Treated area is southwest of media market boundary, inside Philadelphia media market, where volume of political ads is high; control area is northeast of media market boundary, inside New York City media market, where volume of ads is zero.

$T_i$ : Seeing more presidential ads

$Y_i$ : Voter turnout (null)

## Electing an outsider candidates → drug war violence

“Deadly Populism: How Local Political Outsiders Drive Duterte’s War on Drugs in the Philippines” (Ravanilla et al., 2022)



$T_i$ : Electing a candidate outside of the patronage system

$Y_i$ : Government violence in drug war

## Implementing RDD in R

```
library(rdd) # For McCrary's (2008) DCdensity() function
library(rdrobust) # For rdrobust (optimal bandwidth selection & estimation)

# Basic regression discontinuity
my_model <- lm(y ~ treat + rv, data = D)

# Discontinuity with regression lines with different slopes
# on either side of the discontinuity
my_model <- lm(y ~ treat + rv * treat, data = D)

# Regression discontinuity with quadratic functional form
D$rv2 <- D$rv^2 # New variable that is the square of the running variable
my_model <- lm(y ~ treat + rv + rv2, data = D)

# Regression discontinuity with cubic functional form
D$rv2 <- D$rv^2 # Square of the running variable
D$rv3 <- D$rv^3 # Cube of the running variable
my_model <- lm(y ~ treat + rv + rv2 + rv3, data = D)

# Local linear regression discontinuity with optimal bandwidth
# Note: The argument "c = 0" indicates that the cutoff in the
#       running variable occurs when rv = 0
my_model <- rdrobust(D$y, D$rv, c = 0)
```

**Complete the exercise in the R script from the course website...**

## Exercise solutions

```
# Run the McCrary density test to check for sorting on either side of  
# the cutoff  
# Use the function DCdensity() from the R library "rdd"  
# The value that is returned is a p-value. What does it mean?  
DCdensity(D$rv, cutpoint = 0)
```

## Exercise solutions

```
# Variables to measure the square and cube of the running variable
D$rv2 <- D$rv^2
D$rv3 <- D$rv^3

# Replicate TABLE 2 (p. 25) in the article
# Pay attention to the RDD bandwidth and the specification

# Model 1
table_2_model_1 <- lm(dv ~ treat + rv * treat,
                     data = subset(D, margin < 0.05))
summary(table_2_model_1)

# Model 2
table_2_model_2 <- lm(dv ~ treat + rv + rv2 + rv3,
                     data = D)
summary(table_2_model_2)

# Model 3 (these estimates will not be exact replications because
# the author is not using the R implementation of optimal bandwidth)
table_2_model_3 <- rdrobust(D$dv, D$rv, c = 0)
summary(table_2_model_3)
```

## Exercise solutions

```
# Model 4
table_2_model_4 <- lm(dv_win ~ treat + rv * treat,
                     data = subset(D, margin < 0.05))
summary(table_2_model_4)

# Model 5
table_2_model_5 <- lm(dv_win ~ treat + rv + rv2 + rv3, data = D)
summary(table_2_model_5)

# Model 6 (these estimates will not be exact replications because
# the author is not using the R implementation of optimal bandwidth)
table_2_model_6 <- rdrobust(D$dv_win, D$rv, c = 0)
summary(table_2_model_6)
```





## Exercise solutions

```
# extremist share of primary donations
# Variable is called "prim_share"

table_a5_model_2a <- lm(prim_share ~ treat + rv * treat,
  data = subset(D, margin < 0.05))
summary(table_a5_model_2a)
table_a5_model_2b <- rdrobust(D$prim_share, D$rv, c = 0)
summary(table_a5_model_2b)
table_a5_model_2c <- lm(prim_share ~ treat + rv + rv2 +
  rv3, data = D)
summary(table_a5_model_2c)
```

## Exercise solutions

```
# extremist share of primary PAC donations
# Variable is called "prim_pac_share"

table_a5_model_3a <- lm(prim_pac_share ~ treat + rv * treat,
  data = subset(D, margin < 0.05))
summary(table_a5_model_3a)
table_a5_model_3b <- rdrobust(D$prim_pac_share, D$rv, c = 0)
summary(table_a5_model_3b)
table_a5_model_3c <- lm(prim_pac_share ~ treat + rv + rv2 +
  rv3, data = D)
summary(table_a5_model_3c)
```

## Exercise solutions

```
# extremist total primary donations
# Variable is called "prim_total0"

table_a5_model_4a <- lm(prim_total0 ~ treat + rv * treat,
  data = subset(D, margin < 0.05))
summary(table_a5_model_4a)
table_a5_model_4b <- rdrobust(D$prim_total0, D$rv, c = 0)
summary(table_a5_model_4b)
table_a5_model_4c <- lm(prim_total0 ~ treat + rv + rv2 +
  rv3, data = D)
summary(table_a5_model_4c)
```

## Exercise solutions

```
# lagged dw-nom score
# Variable is called "abs_dw_lag"

table_a5_model_5a <- lm(abs_dw_lag ~ treat + rv * treat,
  data = subset(D, margin < 0.05))
summary(table_a5_model_5a)
table_a5_model_5b <- rdrobust(D$abs_dw_lag, D$rv, c = 0)
summary(table_a5_model_5b)
table_a5_model_5c <- lm(abs_dw_lag ~ treat + rv + rv2 +
  rv3, data = D)
summary(table_a5_model_5c)
```

## Exercise solutions

```
# lagged w-nom score (note inclusion of year FEs, see footnote 41)
# Variable is called "abs_lag_wnom"

table_a5_model_6a <- lm(abs_lag_wnom ~ treat + rv * treat +
                        factor(year), data = subset(D, margin < 0.05))
summary(table_a5_model_6a)
table_a5_model_6b <- rdrobust(D$abs_lag_wnom, D$rv, c = 0)
summary(table_a5_model_6b)
table_a5_model_6c <- lm(abs_lag_wnom ~ treat + rv + rv2 +
                        rv3 + factor(year), data = D)
summary(table_a5_model_6c)
```

## Exercise solutions

```
# lagged vote share
# Variable is called "dv_lag"

table_a5_model_7a <- lm(dv_lag ~ treat + rv * treat,
  data = subset(D, margin < 0.05))
summary(table_a5_model_7a)
table_a5_model_7b <- rdrobust(D$dv_lag, D$rv, c = 0)
summary(table_a5_model_7b)
table_a5_model_7c <- lm(dv_lag ~ treat + rv + rv2 +
  rv3, data = D)
summary(table_a5_model_7c)
```

## Exercise solutions

```
# lagged victory
# Variable is called "dv_win_lag"

table_a5_model_8a <- lm(dv_win_lag ~ treat + rv * treat,
  data = subset(D, margin < 0.05))
summary(table_a5_model_8a)
table_a5_model_8b <- rdrobust(D$dv_win_lag, D$rv, c = 0)
summary(table_a5_model_8b)
table_a5_model_8c <- lm(dv_win_lag ~ treat + rv + rv2 +
  rv3, data = D)
summary(table_a5_model_8c)
```